Tanta University — Department of Physics & Mathematics — Faculty of Engineering

Course Title: Complex and Special Functions | Year: 2rd Computer And Control | Code: PM1201
Date: 2010 (2nd term) | Allowed time: 4 hrs | No. of Pages: (2)

Problem number (1) (17 M)

(a) Find all values of: (i) $\sqrt[3]{1+i}$ (ii) $\cosh\sqrt{z} = 0$.

(b) Show that if $f(z) = u(x,y) + iv(x,y)$ **is analytic, then u(x,y) and v(x,y) are harmonics.**

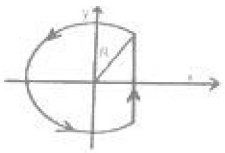
(c) Determine c such that the function is harmonic U = sinx coshcy and find its conjugate harmonic.

Problem number (2) (17 M)

(a) **Evaluate**

(i) $\oint_{|z|=3} (z+1)e^{\frac{1}{z}}\,dz$ (ii) $\oint_{|z|=2} z^2 \sin\frac{2}{z-1}\,dz$ (iii) $\int_C \frac{z^3+1}{(z-1)(z-2)}\,dz$ around $C: |z| = 3$

(b) Using Bromwich contour



To find inverse Laplace transform of $F(s) = \frac{\cosh x\sqrt{s}}{s\cosh\sqrt{s}}$, $0 < x < 1$

c) Find the image of the region $2 \le |z| \le 3$, $\frac{\pi}{6} \le \arg z \le \frac{\pi}{3}$ **by the map** $w = z + \frac{1}{z}$.

Problem number (3) (17 M)

a) Using series solutions to solve the following equations

(ii) $x^2y'' + xy' + (x^2 - \frac{4}{9})y = 0$ near x=0

b) Evaluate the integrations using Gamma and Beta functions

(i) $\int_0^\infty x^3 e^{-2x}\cosh x\,dx$ (ii) $\int_0^{\frac{1}{2}} x^{m-1}\left(\ln\frac{1}{2x}\right)dx$

(iii) $\int_0^{\frac{\pi}{2}} \sqrt{\frac{\sin\theta}{\cos\theta}}\,d\theta$ (v) $\int_0^\infty \frac{1}{1+x^4}\,dx$

Problem number (4) (17 M)

(a) Use Generating function $e^{x(t-\frac{1}{t})} = \sum_{-\infty}^{\infty} J_n(x)t^n$ to prove that:

(i) $e^{ix\sin\theta} = J_0(x) + 2\sum_{n=1}^{\infty} J_{2n}(x)\cos 2n\theta + 2i\sum_{n=0}^{\infty} J_{2n+1}(x)\sin(2n+1)\theta$

(ii) $1 = J_0(x) + 2\sum_{n=1}^{\infty} J_{2n}(x)$ **(iii)** $x = 2\sum_{n=0}^{\infty}(2n+1)J_{2n+1}(x)$

(b) Prove that $J_{\frac{1}{2}} = \sqrt{\frac{2}{\Pi x}} \sin x$, $J_{-\frac{1}{2}} = \sqrt{\frac{2}{\Pi x}} \cos x$ and using these to express $J_{\frac{1}{2}}(x)$, $J_{-\frac{1}{2}}(x)$ in term of sinx and cosx.

(c) Evaluate $\int x^3 J_0 dx$

(17 M)

Problem number (5)

(a) Define and give an example for: fuzzy set, complement of a fuzzy set, union and intersection of two fuzzy sets .

(b) Explain and indicate by examples the deviations between fuzzy sets and ordinary sets .

(c) For the fuzzy subset
A= { (1,0.2) , (2,0.7) , (3,0.6) , (4,0.5), (5,0.8) , (6,1) ,(7,0.4),(8,0.9) } , find ,the height , the core ,the support , the strong α-cut , the weak α-cut (α= 0.4).

(d) If R is a fuzzy relation from A to B and S is a fuzzy relation from B to C . Find RoS , where

R	a	b	c	d
1	0.1	0.2	0.0	1.0
2	0.3	0.3	0.0	0.2
3	0.8	0.9	1.0	0.4

S	α	β	γ
a	0.9	0.0	0.3
b	0.2	1.0	0.8
c	0.8	0.0	0.7
d	0.4	0.2	0.3